Statements and Logical Connectives

Negation is the process through which we change a statement to its opposite meaning. We usually use the word not in negation.

Example:

It is hot outside.

It is not hot outside.

Note: The negation of a true statement is always a false statement (and vice versa).

Quantifiers:

All, some, none (no)

** Use caution when negating statements with quantifiers!

Examples: Negate the following statements.

1. All lakes contain fresh water.
   What is the truth value of this statement?
   False – Salt Lake in Utah

   So the negation must be:
   No lakes contain fresh water. – Still False – NOT CORRECT!
   Not all lakes contain fresh water. True -- CORRECT
   Some lakes contain fresh water. True -- CORRECT

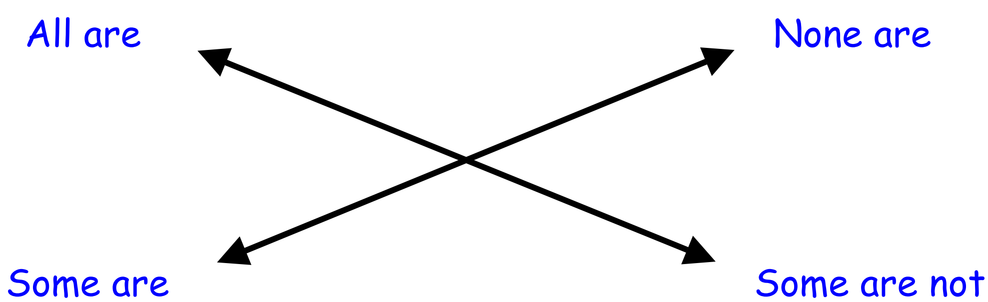
2. Some students are boys.
   What is the truth value of this statement?

True. Therefore its negation must be false.

Some students are not boys. -- Still true – NOT CORRECT!
All students are boys. – CORRECT

Negation of Quantified Statements

Form of statement	**Form of negation**
All are	Some are not
None are	Some are
Some are	None are
Some are not	All are



Statements consisting of two or more simple statements combined using a logical connective are compound statements.

Logical Connectives

And, or, if . . . . then, if and only if, not

Example:

1. The sky is green and it is going to rain.

2. If the phone rings, then you will answer it.

Using Symbols to Represent Statements:

NOT Statements

We use the ~ symbol to represent the negation of a statement.

Example:
Let p = Mark is a piano player.
Then
~p = Mark is not a piano player.

** For any statement p, ~(~p) = p.

- (Think: negative X negative = positive)

We use the letters p, q, r to represent statements that are not negated, and the ~ symbol with p, q, r to represent statements that are negated.

AND Statements (Conjunction)

We use the $\wedge$ symbol to represent "AND".

Example:
1. Let
.p = You are in Math 120.
.q = You will enjoy logic.

Then

.p $\wedge$ q = You are in Math 120 and you will enjoy logic.

2. Let

.p = Bears are big.

.q = Bears are nice.

Then

.p $\wedge$ ~q = Bears are big and bears are not nice.

OR Statements (Disjunction)

We use the $\vee$ symbol to represent "OR".

Example:

1. Let

.p = You will take Math 110.

.q = You will take Math 103.

Then

.p $\vee$ q = You will take Math 103 or you will take Math 110.

2. Find ~p $\vee$ ~q = You will not take Math 103 or you will not take Math 110.

Understanding how commas are used to group statements:

Let

.p = Sue will order meat.

.q = Sue will order soup.

.r = Sue will order salad.

Write the following in symbolic form:

1. Sue will order meat, and soup or salad.

   $p \wedge (q \vee r)$

2. Sue will order meat and soup, or salad.

   $(p \wedge q) \vee r$

3. Sue will order meat, or soup and salad.

   $p \vee (q \wedge r)$

<u>Negating Compound Statements</u>

A negation only negates the statement that immediately follows it. In order to negate a compound statement, we must enclose it in parentheses.

$\sim (p \wedge q)$

<u>Example</u>:

Let

.p = I like math.

.q = I like science.

1. $p \wedge q$ = I like math and I like science.
   Alternative: I like math and science.

2. $\sim (p \wedge q)$ = <u>It is not true that</u> I like math and science.

If – Then Statements (Conditional)

We use the $\rightarrow$ symbol to represent "if . . . then".

.p $\rightarrow$ q means "if p, then q."

p, or the "if" part, is called the antecedent.

.q, or the "then" part, is called the consequent.

Sometimes the "then" is left out of conditional statements.

"Conditional" means that the first part has to happen before the second part can happen.

Example:

Let

.p = you are doing well on your tests.

.q = You will get an A.

Then

1. $p \rightarrow q$ = If you are doing well on your tests, (then) you will get an A.

2. $\sim p \rightarrow \sim q$ = If you are not doing well on your tests, then you will not get an A.

3. $\sim (p \rightarrow q)$ = It is not true that if you are doing well on your tests, you will get an A.

IF AND ONLY IF (Biconditional)

We use $\leftrightarrow$ to represent "if and only if", also abbreviated as "iff".

Example:

Let

.p = It is cloudy.

.q = It is raining.

1. $p \leftrightarrow q$ = It is cloudy if and only if it is raining.

2. $\sim(p \leftrightarrow \sim q)$ = It is not true that it is cloudy if and only if it is not raining.

Evaluate First

Order of Operations (in Logic)

1. Negation, $\sim$
2. Conjunction, $\wedge$; Disjunction, $\vee$
3. Conditional, $\rightarrow$
4. Biconditional, $\leftrightarrow$

Evaluate Last

Examples: pages 93 – 95

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Let p = Firemen work hard.

.q = Firemen wear red suspenders

$\sim p \leftrightarrow \sim q$ = Firemen do not work hard if and only if they (firemen) do not wear red suspenders.

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Let p = The water is 70°.

q = The sun is shining.

r = We go swimming.

$q \rightarrow (p \leftrightarrow r)$ = If the sun is shining, then we will go swimming if and only if the water is 70°.