

## Inductive Reasoning

Inductive reasoning or induction is the process of reasoning to a general conclusion through observation of specific cases.

Inductive reasoning is often used by mathematicians and scientists to predict answers to complicated problems. For this reason, inductive reasoning is part of the scientific method. When a scientist or mathematician makes a prediction based on specific observations, it is called a hypothesis or conjecture.

Examples: Use inductive reasoning to predict the next three numbers in the pattern.

1. 3, 6, 12, 24, 48, 96, 192. (multiplying by 2)
2. 1, 4, 9, 16, 25, 36, 49. (squares of 1, 2, 3, ...)
3.  $a + 1$ ,  $a + 3$ ,  $a + 5$ ,  $a + 7$ ,  $a + 9$ ,  $a + 11$ . (adding 2)
4. 4, 5.5, 7, 8.5, 10, 11.5, 13. (adding 1.5)
5. 9, 7, 10, 8, 11, 9, 12, 10, 13, 11. (subtract 2, add 3)
6. A special sequence is called the Fibonacci sequence. It is named after Leonardo Fibonacci from Italy who presented it in 1201. This sequence is very intriguing because the numbers in the sequence are often found in nature. The first six terms of the sequence are listed below. Give the next six numbers in the sequence. 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144.

(The next term is the sum of the previous two terms.)

When forming a general conclusion using inductive reasoning, you should test your hypothesis with several special cases to see whether your conjecture is correct. If a special case is found that satisfies the conditions of the conjecture, but produces a different result, the case is called a counterexample. The counterexample proves that your hypothesis is false.

Deductive reasoning is the process of reasoning to a specific conclusion from a general statement. Mathematicians use deductive reasoning to prove conjectures true or false.

Example:

Pick any number and multiply the number by 6. Add 3 to the product. Divide the sum by 3 and subtract 1 from the quotient.

i.e. I will choose the number 4.

So,  $4 \times 6 = 24$ .

$24 + 3 = 27$

27 divided by 3 = 9

$9 - 1 = 8$ .

My final answer is 8.

a) What is the relationship between the number you started with and the final answer?

My final answer is twice my original number.

b) Arbitrarily select some different numbers and repeat the process, recording the original number and the results.

Original Number	Manipulations	Result

c) Can you make a conjecture about the relationship between the original number and the final number?

The result will always be twice the original number.

d) Try to prove, using deductive reasoning, the conjecture you made in part c.

Let  $n$  be my original number that I will manipulate.

Multiply by 6:

$$6 \times n = 6n$$

Then add 3:

$$6n + 3.$$

Divide this sum by 3:

$$\frac{6n+3}{3} = \frac{6n}{3} + \frac{3}{3} = 2n+1$$

(Because we must divide each term by 3)

Then subtract 1:

$$\begin{aligned}(2n + 1) - 1 &= 2n + 1 - 1 \\ &= 2n\end{aligned}$$

Thus, my result is  $2n$ , which is twice my original number.