Properties of Exponents

I. <u>The Product Rule</u>

Multiplying with like bases:

For any number a and any positive integers m and n,

$$a^m \bullet a^n = a^{m+n}$$

i.e. When multiplying numbers with powers, if the bases are the same, keep the base and add the exponents.

<u>Examples</u>: Multiply and simplify. Leave the answer in exponential notation.

1. pg. 54 # 2 in <u>Intermediate Algebra</u> by Bittinger

$$7^3 \bullet 7^4 = 7^{3+4} = \boxed{7^7}$$

2. pg. 54 # 8 in <u>Intermediate Algebra</u> by Bittinger

$$4a^3 \bullet 2a^7 = (4 \bullet 2)a^{3+7} = \boxed{8a^{10}}$$

3. pg. 54 # 12 in <u>Intermediate Algebra</u> by Bittinger

$$\left(m^6 n^5\right)\left(m^4 n^7 p^0\right) = m^{6+4} n^{5+7} p^0 = \boxed{m^{10} n^{12}}$$

Remember: Any number raised to the zero power equals 1.

II. The Quotient Rule

Dividing with like bases:

For any nonzero number a and any positive integers m and n, where $m > n$,

$$\frac{a^m}{a^n} = a^{m-n}$$

i.e. When dividing numbers with powers, if the bases are the same, keep the base and subtract the exponent of the denominator (bottom) from the exponent of the numerator (top).

<u>Examples</u>: Divide and simplify.

1. pg. 54 # 16 in <u>Intermediate Algebra</u> by Bittinger

$$\frac{20a^{20}}{5a^4} = \boxed{4a^{16}}$$

2. pg. 54 # 22 in <u>Intermediate Algebra</u> by Bittinger

$$\frac{18x^8y^6z^7}{-3x^2y^3z} = \left(\frac{18}{-3}\right)x^{8-2}y^{6-3}z^{7-1} = \boxed{-6x^6y^3z^6}$$

III. <u>Negative Exponents</u>

For any real number a that is a nonzero number and any integer n,

$$a^{-n} = \frac{1}{a^n}$$

i.e. The numbers a^{-n} and a^n are reciprocals of each other.

<u>Examples</u>: Write an equivalent expression without negative exponents.

1. $x^{-12} = \boxed{\frac{1}{x^{12}}}$

2. $4m^{-5} = \boxed{\frac{4}{m^5}}$

3. $\frac{m^{-12}y^7}{n^{-8}} = \boxed{\frac{n^8y^7}{m^{12}}}$

IV. Factors and Negative Exponents

For any nonzero real numbers a and b and any integers m and n,

$$\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}$$

i.e. Any factor can be moved to the other side of the fraction bar if the sign of the exponent is changed.

<u>Examples</u>: Write an equivalent expression without negative exponents, and, if possible, simplify.

1. pg. 55 # 106 in <u>Intermediate Algebra</u> by Bittinger

$$\left(\frac{2x^3y^{-2}}{3y^{-3}}\right)^3 = \left(\frac{2x^3y^3}{3y^2}\right)^3 = \left(\frac{2x^3}{3}y^{3-2}\right)^3 = \boxed{\left(\frac{2x^3y}{3}\right)^3}$$

In the next section, I will show you how to raise a power to a power, but for now we will leave it like this.

2. modified version of pg. 55 # 108 in Intermediate Algebra by Bittinger

$$\left(\frac{30x^5y^{-7}}{6x^{-2}y^{-6}}\right)^{-3} = \left(\frac{30x^5x^2y^6}{6y^7}\right)^{-3} = \left(\frac{30}{6}\bullet x^{5+2}y^{6-7}\right)^{-3} = \left(5x^7y^{-1}\right)^{-3} = \boxed{\left(\frac{5x^7}{y}\right)^{-3}}$$

V. The Power Rule

Raising a Power to Power:

For any real number a and any integers m and n,

$$\left(a^m\right)^n = a^{mn}$$

i.e. To raise a power to a power, simply multiply the exponents.

Examples: Simplify the two problems from the last section.

1. pg. 55 # 106 in Intermediate Algebra by Bittinger

$$\left(\frac{2x^3y^{-2}}{3y^{-3}}\right)^3 = \left(\frac{2x^3y}{3}\right)^3 = \frac{(2)^3\left(x^3\right)^3(y)^3}{(3)^3} = \boxed{\frac{8x^9y^3}{27}}$$

2. modified version of pg. 55 # 108 in Intermediate Algebra by Bittinger

$$\left(\frac{30x^5y^{-7}}{6x^{-2}y^{-6}}\right)^{-3} = \left(\frac{5x^7}{y}\right)^{-3} = \frac{(5)^{-3}\left(x^7\right)^{-3}}{(y)^{-3}} = \frac{(y)^3}{(5)^3\left(x^7\right)^3} = \boxed{\frac{y^3}{125x^{21}}}$$

Note: There is a real nice table detailing all of the definitions and properties of exponents on page 54 of the Intermediate Algebra:Sixth Edition book by Bittinger.